Chapter 1

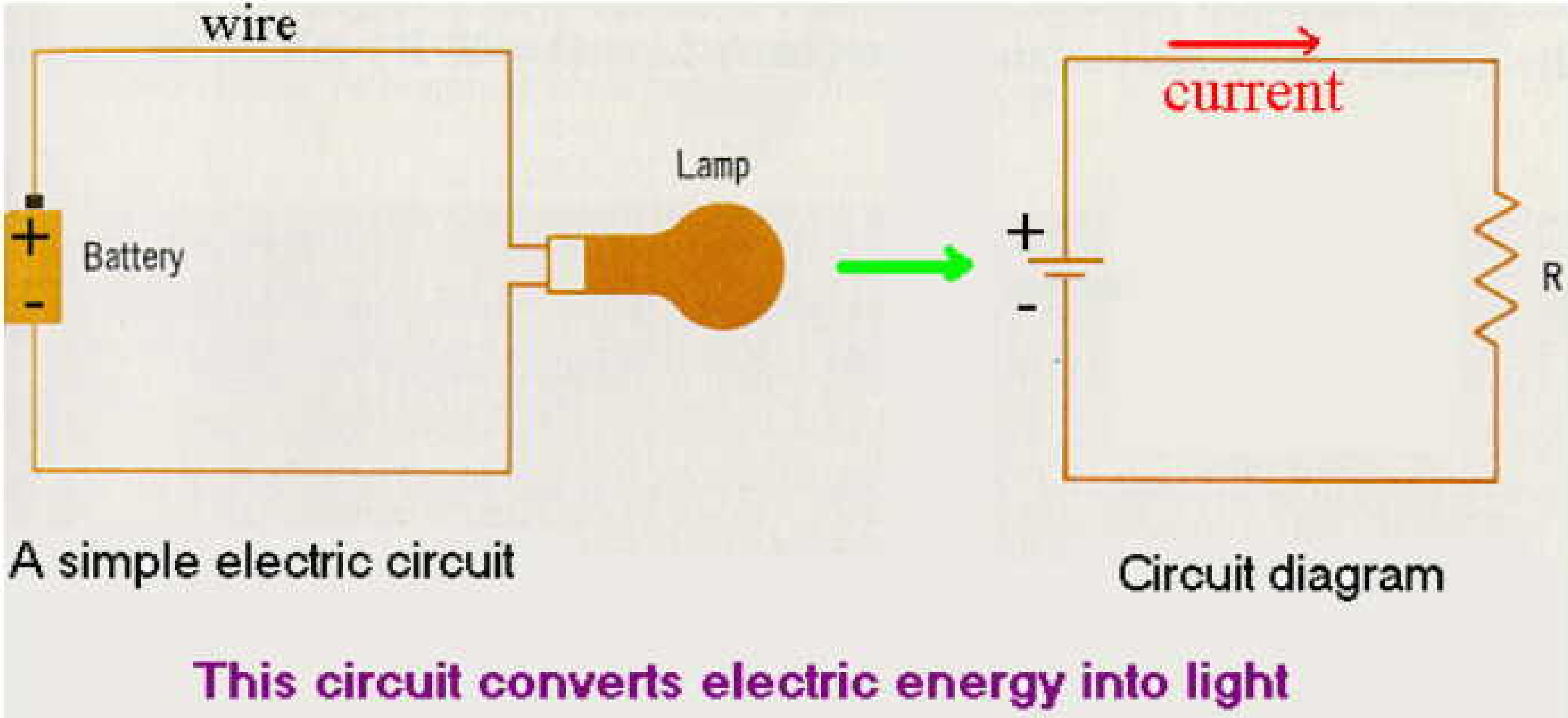
Electric Circuit Variables

Early History of Electrical Science

Self study

Preliminary Definitions

Electricity is the physical phenomenon arising from the existence and interaction of electric charge. An electric circuit is an interconnection of electrical elements linked together in a closed path so that an electric current may flow continuously in order to manipulate electric energy.

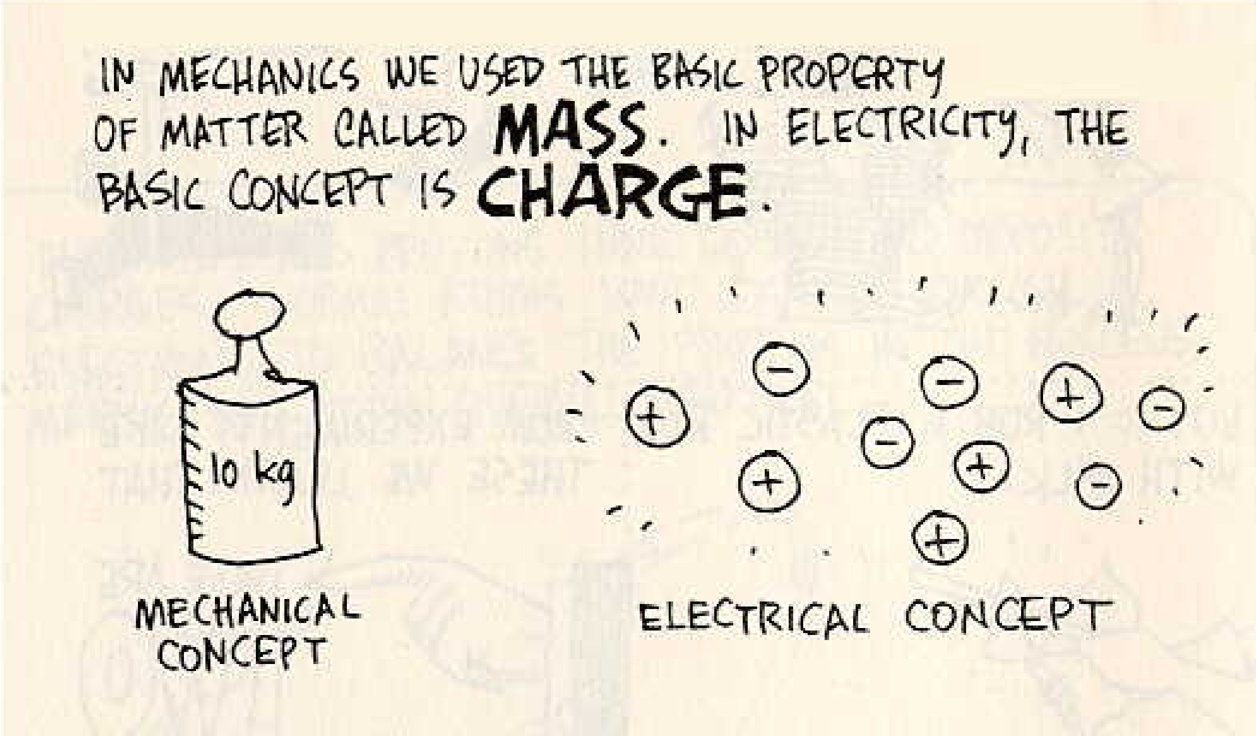


Preliminary Definitions (cont.)

Engineering combines the study of mathematics and natural and social sciences to direct the forces of nature for the benefit of humankind.

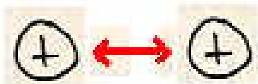
Electrical Engineering deals with the design and production of devices (often called hardware) based on electric circuits.

The outstanding characteristics of electricity when compared with other energy sources are its mobility and flexibility. Electrical energy can be moved to any point along a couple of wires and can be converted to light, heat or mechanical force.



In the late 1740s, Benjamin Franklin developed the theory that there are two kinds of charge, positive and negative.

Unlike charges attract



Like charges repel

Electric Current

Electric current is defined in terms of two quantities: **Direction and magnitue.**

The direction of flow of electric current is conventionally represented as the direction of flow of POSITIVE charge.

$$\oplus \longrightarrow$$

This convention was initiated by Benjamin Franklin and is commonly used today in most textbooks and in practice.

But we know that electrons (with negative charge) are resposible for electric currents in metal conductors (e.g., in copper wires). Therefore, and by convention, we will assign the direction of current as opposite to the direction of flow of electrons.

Electric Current

Electric current is defined in terms of two quantities: **Direction and magnitue.**

The magnitude of current is given by the rate of change of charge past a given point in an electric circuit:

$$\boxed{i(t) = \frac{dq}{dt}}$$

Andre-Marie Ampere (1775-1836), a French mathematician and physicist, defined the electric current (in the 1820s) and developed a way to measure it. He was honored by having the unit of electric current, the ampere, named after him in 1881.

Electric Current

$$\boxed{i(t) = \frac{dq}{dt}}$$

The symbol for current is i .

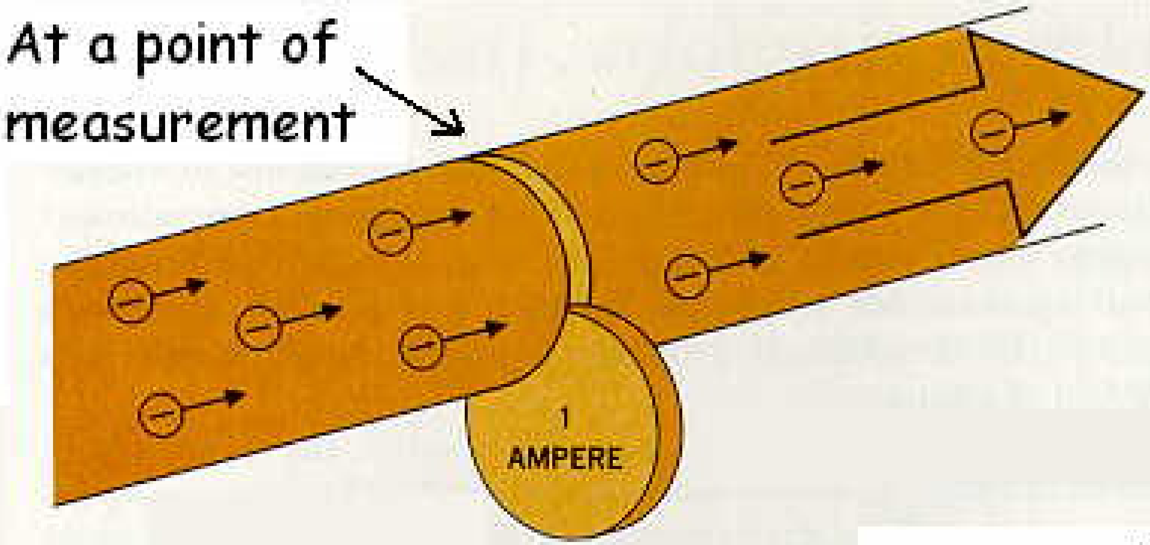
The unit of current is ampere (A); an ampere is 1 coulomb per second or 1 C/s.

The symbol for charge is q.
Its unit is the coulomb (C).

The charge on one electron is $-1.602\text{x}10^{-19}$ C.

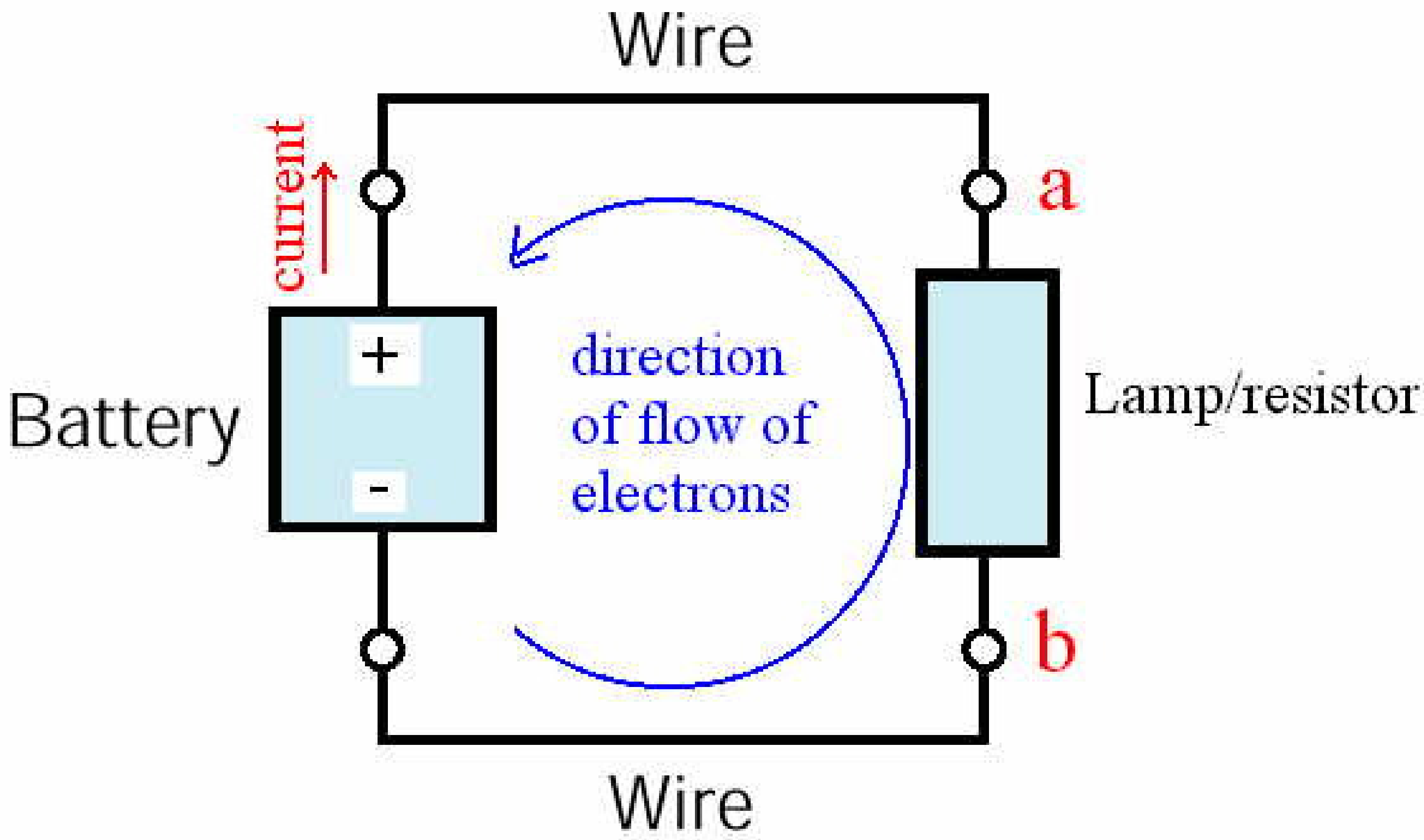
Thus -1 C is the charge of $6.24 \text{ x } 10^{18}$ electrons

Electric Current in a wire



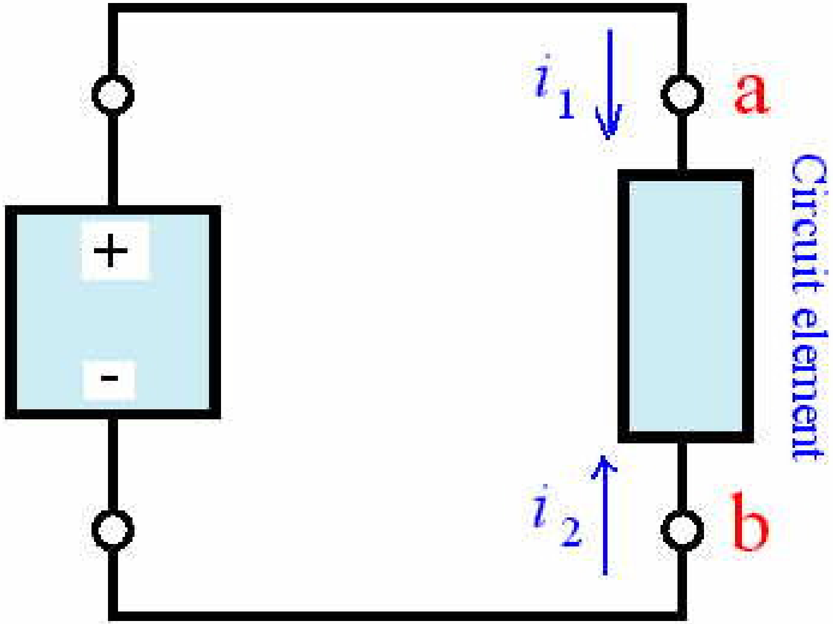
1 coulomb per second = 1 ampere = 6.24×10^{18} electrons per second

Electric Current



Electric Current
in a circuit element

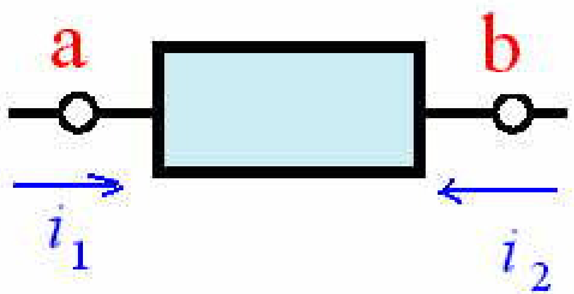
How do we label the direction of electric current in a circuit element?



Does not matter which direction is assigned on the diagram.

Note that $i_1 = -i_2$

Electric Current
in a circuit element



Terminology:

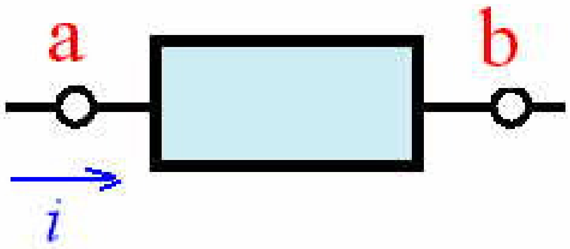
The current i_1 is the rate of flow of electric charge from terminal a to terminal b.

The current i_2 is the rate of flow of electric charge from terminal b to terminal a.

Note that $i_1 = - i_2$

Electric Current

Examples



Find the charge that has entered terminal a of an element by t time when the current is

$$i = Mt, \quad t \geq 0$$

Solution:

$$i(t) = \frac{dq}{dt} \quad or \quad dq = i(\tau)d\tau$$

Integrating, we get $\displaystyle\int_{q(0)}^{q(t)} dq = \int_0^t i(\tau)d\tau$ or $\displaystyle q(t) - q(0) = \int_0^t i(\tau)d\tau$

$$\boxed{q(t) = \int_0^t i(\tau)d\tau + q(0)}$$

Electric Current

Examples

Find the charge that has entered terminal a of an element by t time when the current is

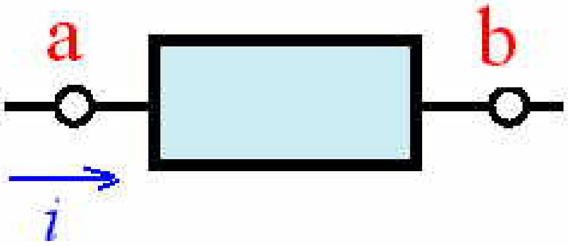
$$i = Mt, \quad t \geq 0 \qquad \text{Assume } q(0) = 0.$$

Solution: (cont.)

$$q(t) = \int_0^t i(\tau)d\tau + \underbrace{q(0)}_{0} = \int_0^t M\tau d\tau = M\frac{t^2}{2} \quad \mathrm{C}$$

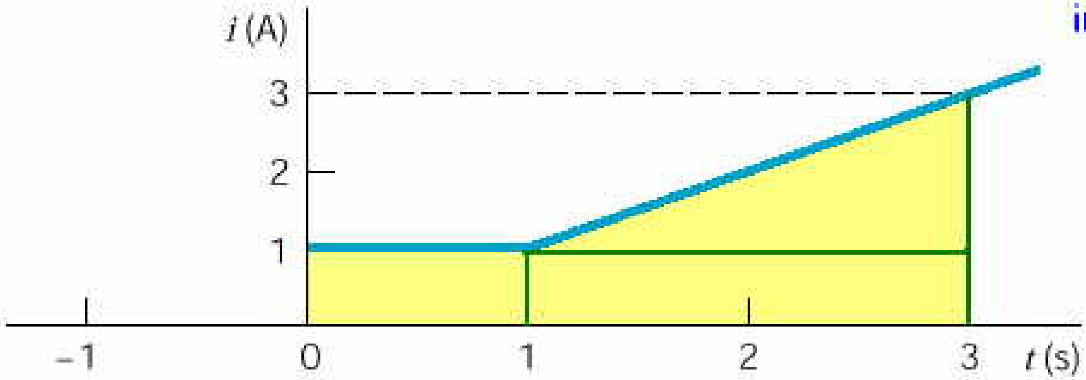
Electric Current

Examples



Find the charge that has entered terminal a of an element from $t = 0$ to $t = 3\text{s}$ when the current is

(i.e., discard initial charge) $q(0) = 0$



Solution: $$q(3) = \int_0^3 i(t)dt + q(0) = \int_0^1 (1)dt + \int_1^3 t\,dt + 0 = t\Big|_0^1 + \frac{t^2}{2}\Big|_1^3 = 5\text{C}$$

Alternatively:

$$q(3) = \int_0^3 i(t)dt + q(0) = \text{Area under curve}(0 < t < 3) + 0$$

$$= (1)(1) + (1)(2) + (2)(2)/2 = 5\text{C}$$

Systems of Units

SI Base Units

Quantity	SI Unit Name	SI Unit Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Systems of Units

Derived Units in SI

Quantity	Unit Name	Formula	Symbol
Acceleration—linear	meter per second per second	m/s^2	
Velocity—linear	meter per second	m/s	
Frequency	hertz	s^{-1}	Hz
Force	newton	$kg \cdot m/s^2$	N
Pressure or stress	pascal	N/m^2	Pa
Density	kilogram per cubic meter	kg/m^3	
Energy or work	joule	$N \cdot m$	J
Power	watt	J/s	W
Electric charge	coulomb	$A \cdot s$	C
Electric potential	volt	W/A	V
Electric resistance	ohm	V/A	Ω
Electric conductance	siemens	A/V	S
Electric capacitance	farad	C/V	F
Magnetic flux	weber	$V \cdot s$	Wb
Inductance	henry	Wb/A	H

Systems of Units

SI Prefixes

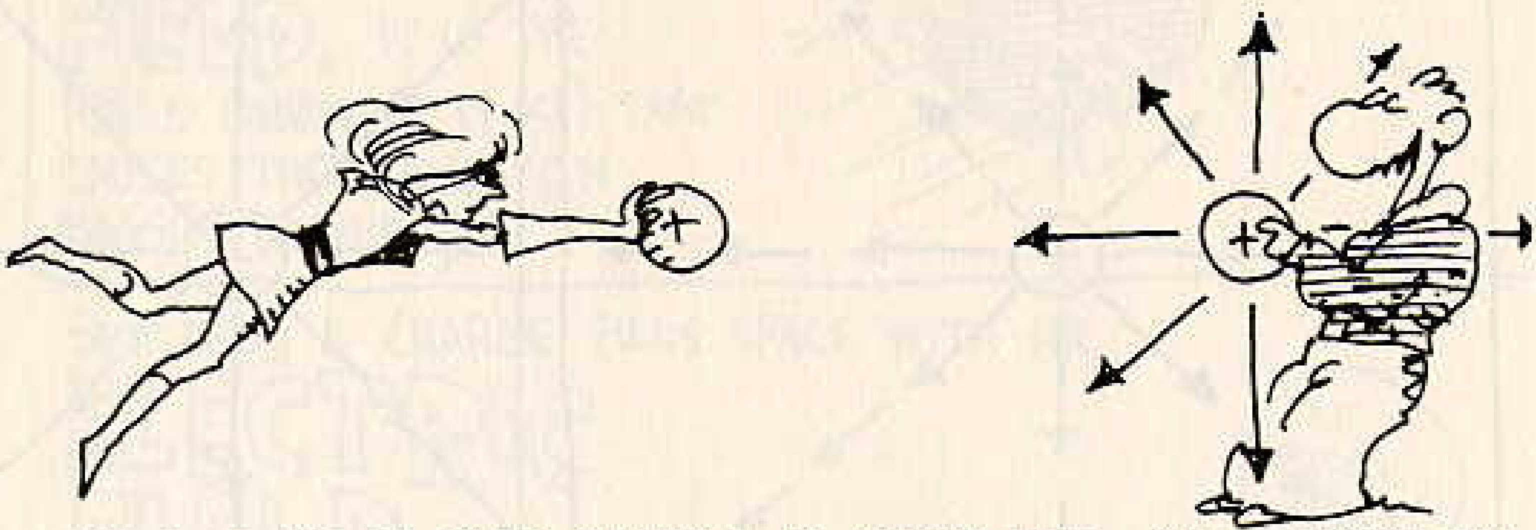
Multiple	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Voltage

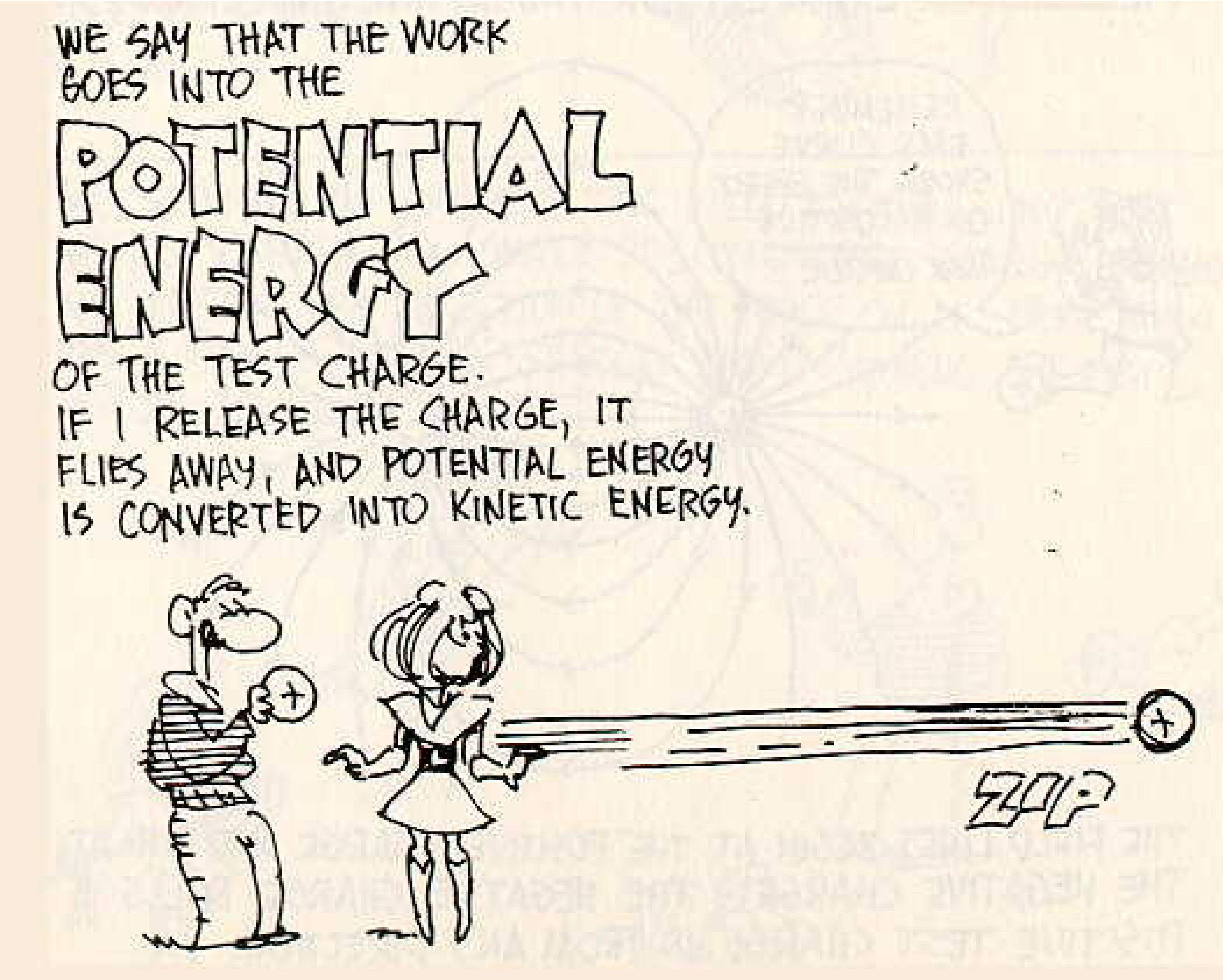
The basic variables in an electric circuit are current and voltage.

Voltage describes the energy required to cause a charge to move across the terminals of a circuit element.

HERE RINGO HOLDS A POSITIVE CHARGE, AND, STARTING FAR AWAY, I BRING A SMALL POSITIVE TEST CHARGE IN CLOSE TO IT.

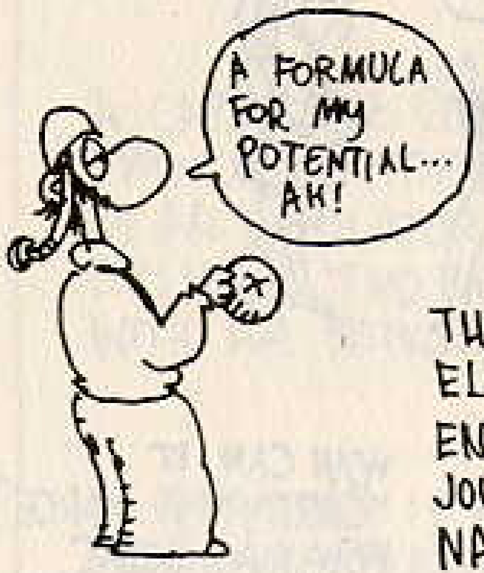


AS I MOVE IN, THE CHARGE IS REPELLED, SO I HAVE TO EXERT FORCE TO PUSH IT CLOSER. FORCE TIMES DISTANCE EQUALS **WORK** — I DO WORK ON THE TEST CHARGE.


WE SAY THAT THE WORK GOES INTO THE
POTENTIAL ENERGY
OF THE TEST CHARGE.
IF I RELEASE THE CHARGE, IT FLIES AWAY, AND POTENTIAL ENERGY IS CONVERTED INTO KINETIC ENERGY.
ZIP

WE WOULD LIKE TO ATTRIBUTE THE POTENTIAL ENERGY SOLELY TO THE ELECTRIC FIELD OF RINGO'S CHARGE, SO WE DIVIDE OUT MY TEST CHARGE AND WRITE:

$$\text{Potential} = \frac{\text{POTENTIAL ENERGY}}{\text{CHARGE}}$$



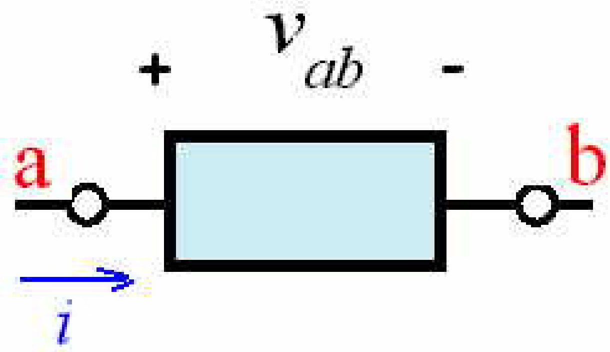
THIS EQUATION DEFINES A NEW QUANTITY, THE ELECTRIC POTENTIAL.* POTENTIAL MEASURES ENERGY PER CHARGE. ITS UNITS ARE JOULES PER COULOMB, WHICH WE GIVE A NAME ALL ITS OWN, THE VOLT.

$$1\ \text{Volt} = 1\ \frac{\text{JOULE}}{\text{COULOMB}}$$

Voltage

When a current flows through a circuit element, it develops a voltage drop across the terminals of that element.

Voltage describes the energy required to cause a charge to move across the terminals of a circuit element.

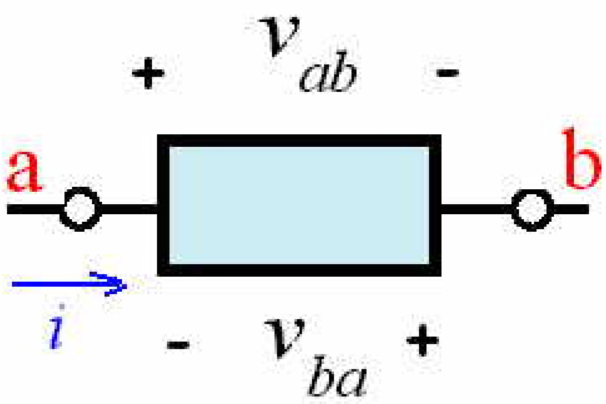


v_{ab} can be read as "the voltage at terminal a with respect to the volatge at terminal b."

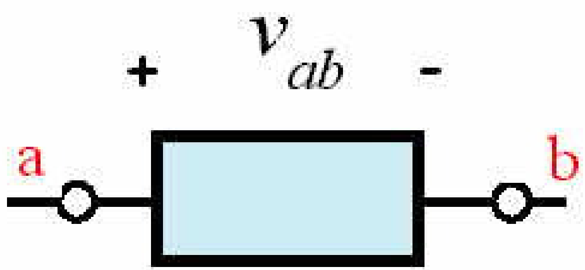
Voltage

The direction of voltage is given by its polarity. Voltage can be positive or negative. The figure below shows two ways of defining the voltage across the element. Here,

$$\boxed{v_{ab} = - v_{ba}}$$



Voltage



The volatge across an element is the work (energy w) required to move a unit positive charge from the - terminal to the + terminal. The unit of voltage is the Volt, V.

$$v = \frac{dw}{dq}$$

When work is linear in the formula reduces to: (here, Q is total charge and w is total work).

$$v = \frac{w}{Q}$$

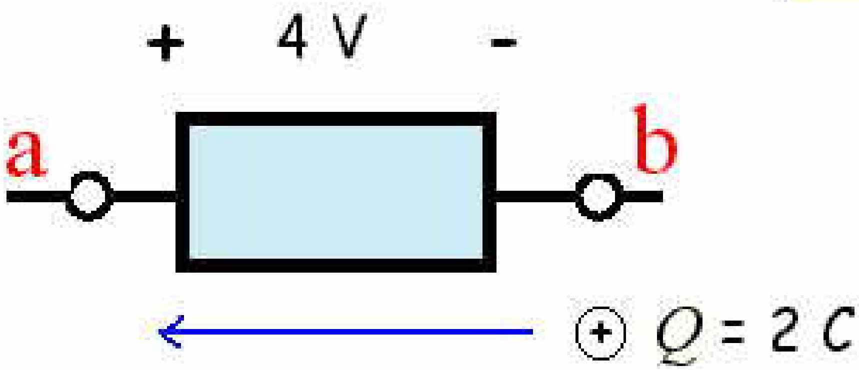
volt = joule/coulomb or V = J/C

The above definition is also valid for "... to move a unit negative charge from the + terminal to the - terminal."

Voltage

Examples

$$v = \frac{w}{Q}$$



Find the energy required to move 2 coulombs of charge through a constant voltage of 4 volts.

Solution:

energy = (voltage) x (total charge) = 4x2 = 8 J

Voltage

Examples

$$v = \frac{w}{Q}$$

The average current in a typical lightning thunderbolt is 20 kA and its typical duration is 0.1s. The voltage between the clouds and the ground is 500 MV. Determine the total charge transmitted to earth and the energy released.

Solution:

The total charge is

$$Q = I.\Delta t = 20{,}000\text{x}0.1 = 2000\text{ C}$$

The total energy released is

$$w = Q.v = 2000\text{x}(500\text{x}10^{6}) = 10^{12}\text{ J} = 1\text{ TJ}$$

Power and Energy

Power is the time rate of expending or absorbing energy.

$$\boxed{p(t) = \frac{dw}{dt}}$$

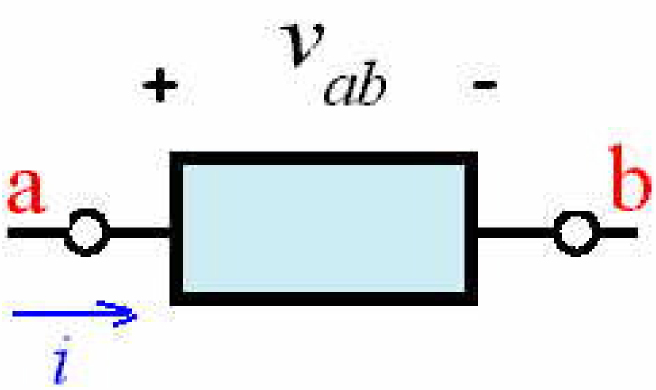
where p is power in watts, w is energy in joules, and t is time in seconds.

Power and Energy

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v.i$$

$$\boxed{p = v.i}$$

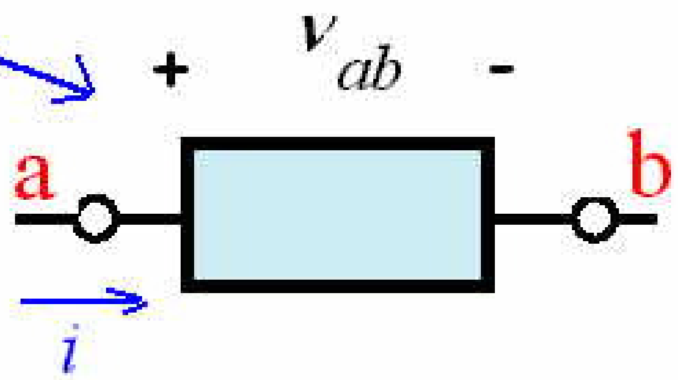
Units W = V.A



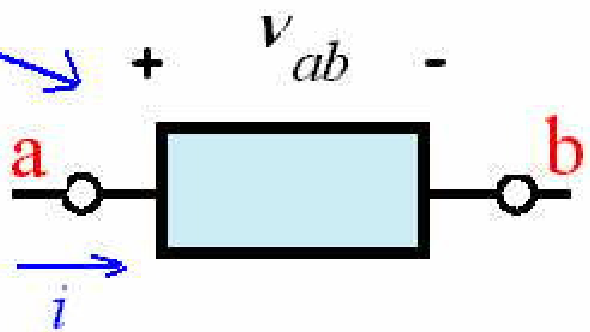
How do we know if a circuit element is delivering power or absorbing power?

The sign of p determines that. But for that to work, we need to use the so called "passive convention."

The assigned direction of the current is directed from the + terminal of the voltage to the - terminal.



The assigned direction of the current is directed from the + terminal of the voltage to the - terminal.



With this convention, $p = v.i$ is the power *absorbed* by the element if $p > 0$.

If $p < 0$, then the power is *delivered* by the element.

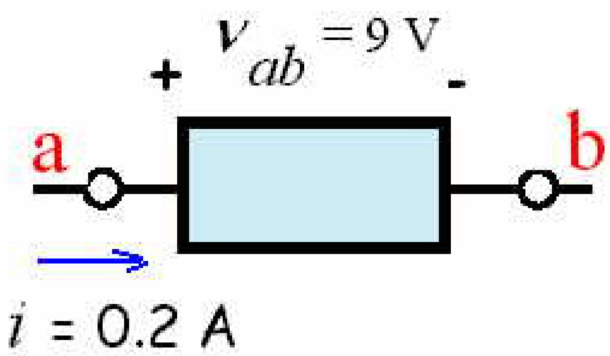
Terminology:

Power absorbed is also called "power dissipated by the element" or" power delivered to the element."

"Power delivered by the element" is also be called "power supplied by the element."

Power/Energy Examples

Consider the element shown below. Find the power and tell if it is actually absorbed by the element *or* delivered by the element.



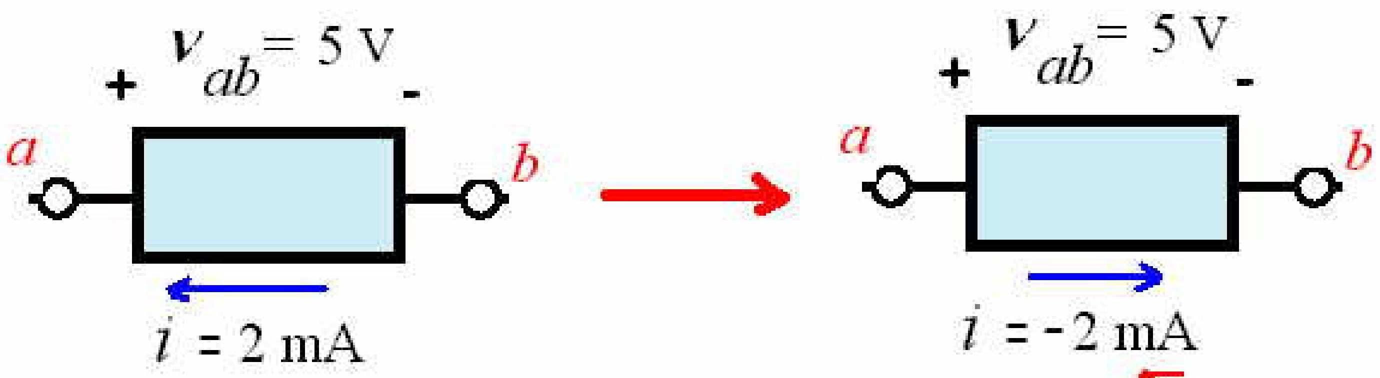
Solution:

Since current direction and volatge polarity adheres to the passive convention, then $p = 9 x 0.2 = 1.8$ W

P is positive, therefore power is absorbed by the element.

Power/Energy Examples

Consider the element shown below. Find the power and tell if it is actually absorbed by the element *or* delivered by the element.



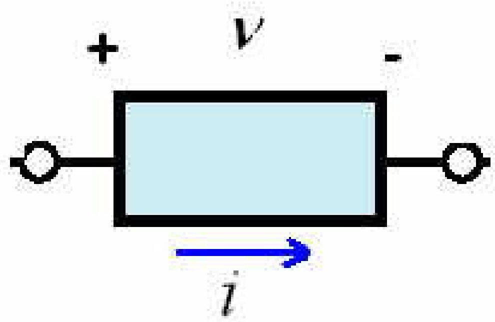
Solution:

Since current direction and volatge polarity does not adhere to the passive convention, *we first reverse the direction of the current and change its sign.* Passive convention is now met:

$$p = 5\text{x}(-0.002) = -0.01 \text{ W}$$

$p < 0$, therefore power is delivered (supplied) by the element.

Power/Energy Examples



Consider the circuit element shown in the following figure with

$$v = -\,8e^{-t}\ \text{V} \quad \text{and} \quad i = 20e^{-t}\ \text{A}$$

Assume the current and voltage to be zero for $t < 0$. Find the energy supplied by this element at $t = 1$ s.

Solution:

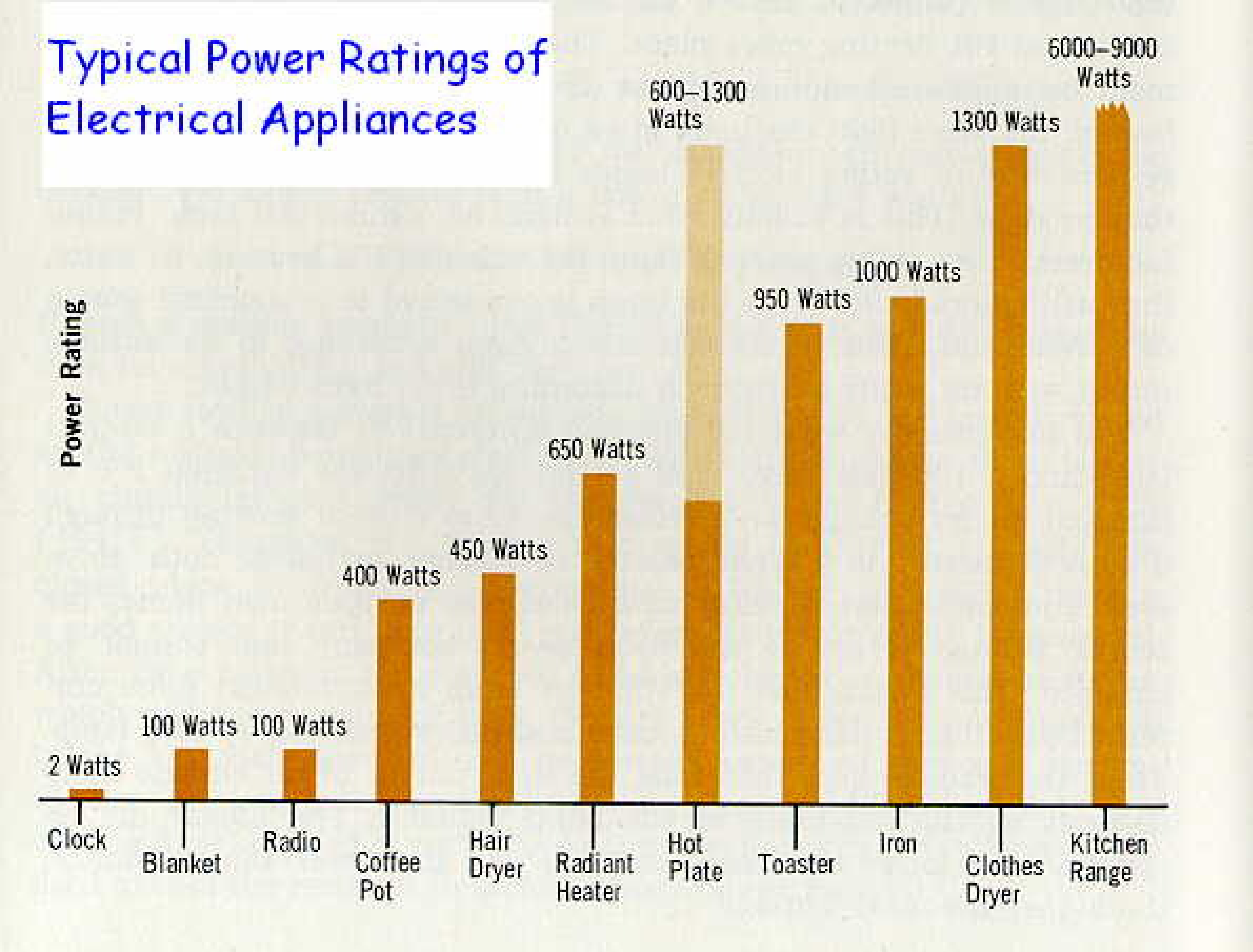
$$p = vi = (-\,8e^{-t})(20e^{-t}) = -160\,e^{-2t}\ \text{W}$$

This element is providing energy to the charge flowing through it.

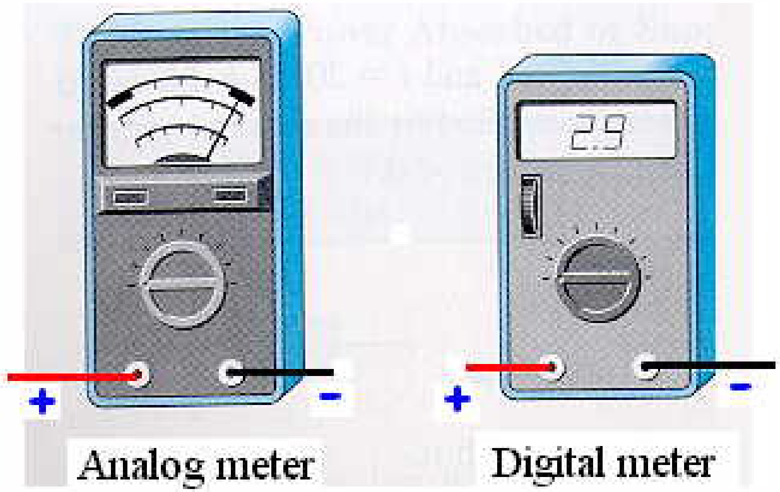
Energy is the integral if power:

$$\boxed{w(t) = \int_0^t p\,d\tau} = \int_0^t -160e^{-2\tau}d\tau = -160\left.\frac{e^{-2\tau}}{-2}\right|_0^t = 80(\,e^{-2t} - 1\,)\ \text{J}$$

$w(1) = -69.2$ J (negative sign means energy supplied to the charges)


Typical Power Ratings of Electrical Appliances
Power Rating
2 Watts
100 Watts
100 Watts
400 Watts
450 Watts
650 Watts
600–1300 Watts
950 Watts
1000 Watts
1300 Watts
6000–9000 Watts
Clock
Blanket
Radio
Coffee Pot
Hair Dryer
Radiant Heater
Hot Plate
Toaster
Iron
Clothes Dryer
Kitchen Range

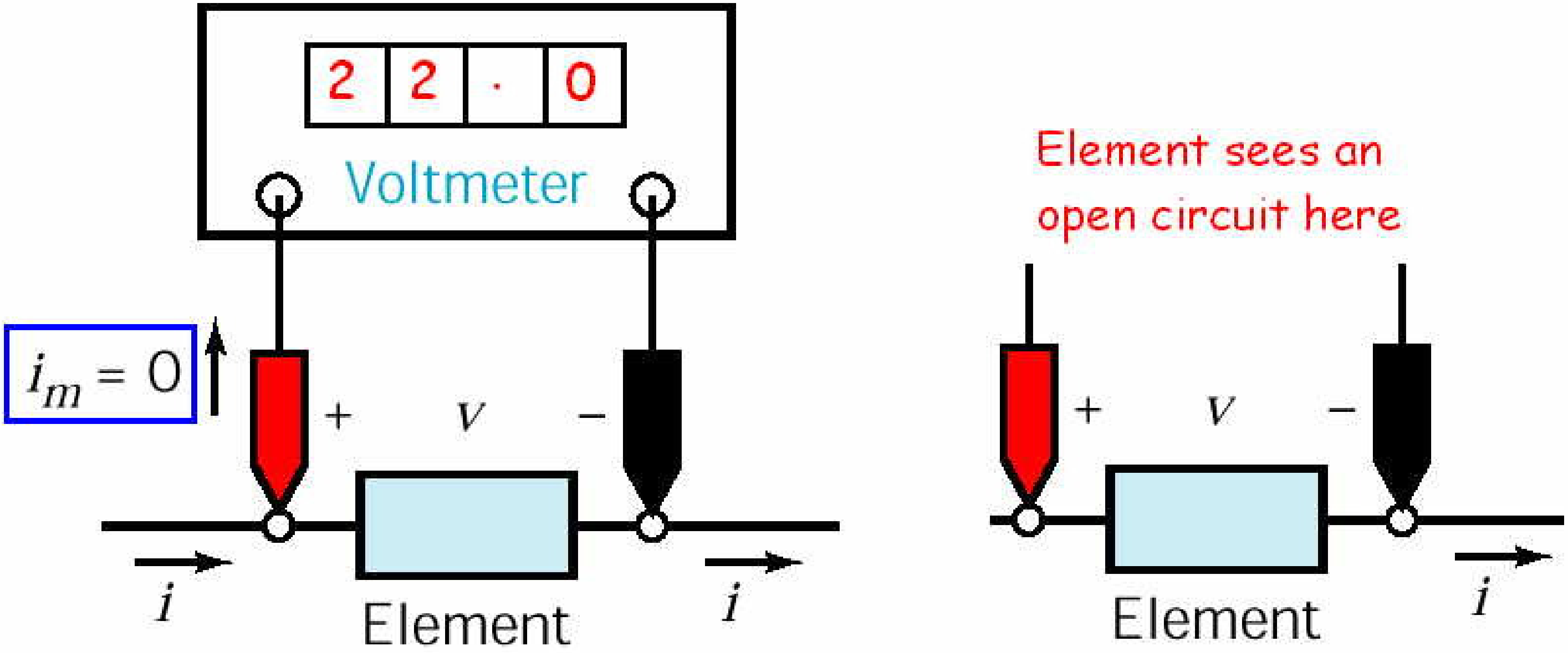
Voltmeters and Ammeters



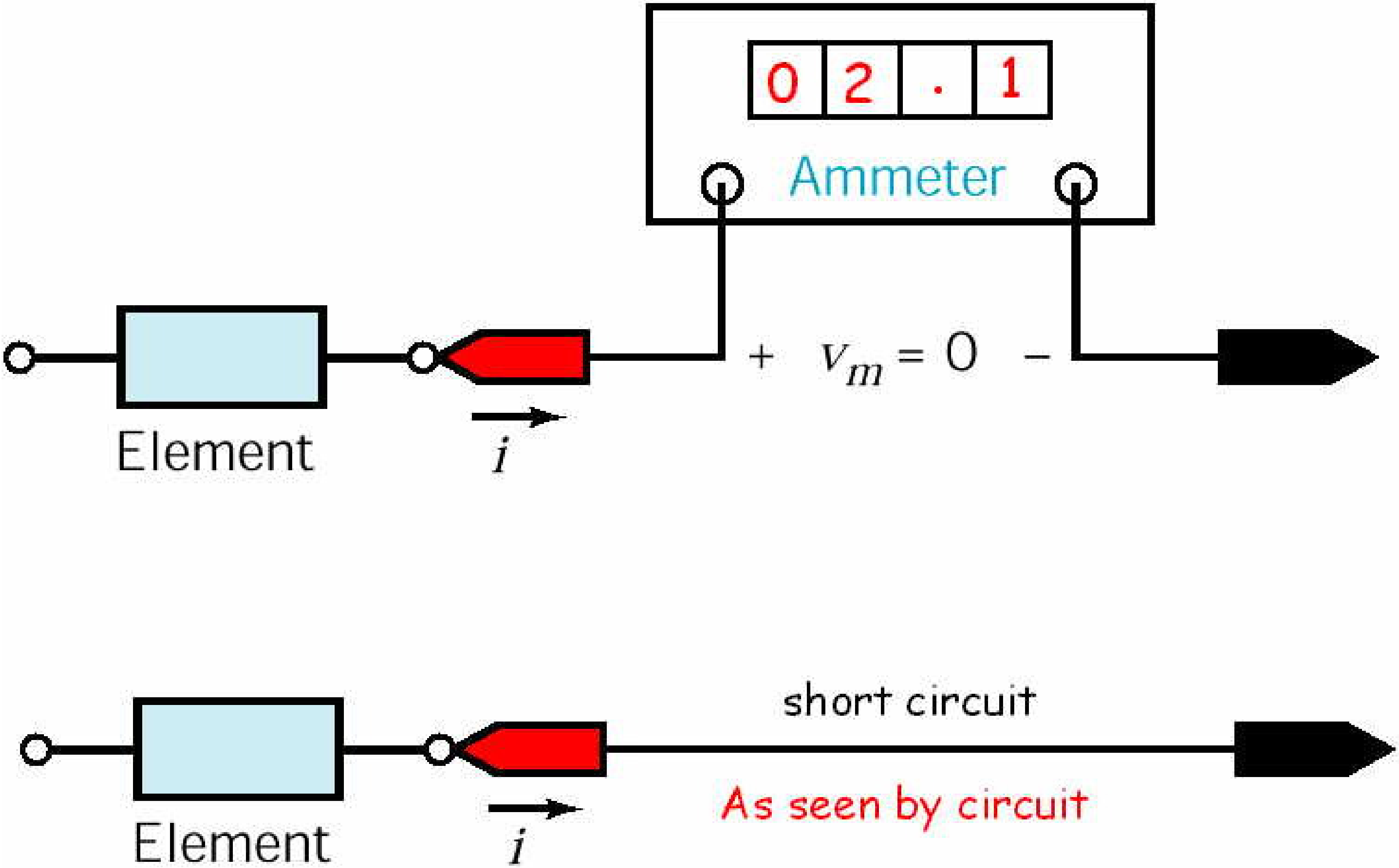
An ammeter is an instrument used to measure current. It has zero voltage across its terminals; when used in a circuit, it is seen as a short circuit (i.e, it acts like a conducting wire).

A voltmeter is an instrument used to measure voltage across an element in a circuit. It has a terminal current equal to zero. It acts as an open circuit.

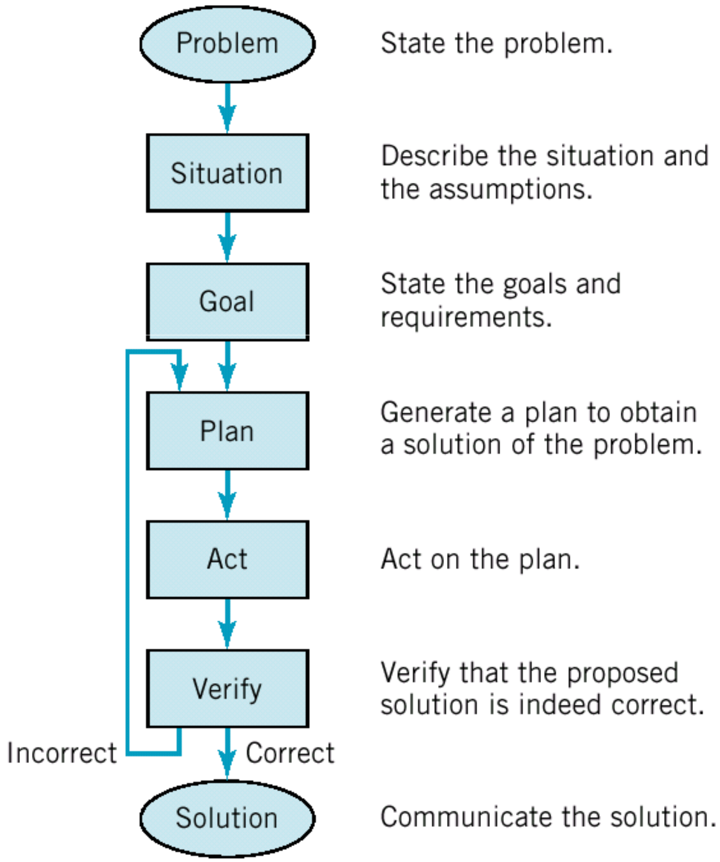
Measuring Voltage



Measuring Current



The problem-solving method.



Summary

- Early History of Electrical Science
- Electric Current and Current Flow
- Systems of Units
- Voltage
- Power & Energy
- Voltmeters and Ammeters
- Circuit Analysis and Design